



YOUR GUIDE TO

Language in the Mathematical Practices



[SPEAKAGENT.COM](https://www.speakagent.com)

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Welcome! This guide highlights how the Math+LanguageSM supplemental program by Speak Agent supports the eight Standards for Mathematical Practice.

The Mathematical Practices

MP1: Make sense of problems & persevere in solving them.

- Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.
- They analyze givens, constraints, relationships, and goals.
- They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- They monitor and evaluate their progress and change course if necessary.
- Younger students may rely on using concrete objects or pictures to help conceptualize and solve a problem, whereas older students should be able to explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP1: How Speak Agent Helps

Our Math+Language programs build math literacy skills so that learners can comprehend, contextualize, and communicate mathematical information, problems, and reasoning. Without the academic language of math, especially math reading skills, a learner will find it difficult to adhere to this practice because they may not understand what a question is asking, understand what information is needed to solve a problem, be able to compose explanations, communicate their thinking, or compare problem-solving approaches.

Math+Language helps students acquire the math language to engage in this critical practice through activities that use all six communication modes (listening, speaking, reading, writing, viewing and representing)—especially math reading—with authentic math stories and situations. Math+Language also uses stories that model the process of persevering through problem solving.

☆ [See footnote](#) for examples.





MP2: Reason abstractly and quantitatively.


- Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two abilities to bear:
 - a. the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols;
 - b. the ability to contextualize—to pause as needed during the manipulation process in order to probe into what the symbols mean.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP2: How Speak Agent Helps











Our Math+Language programs provide a variety of activities that take students through the problem solving process. These activities provide different levels of independent reasoning and learning scaffolds along the way using activities such as *Explain Your Work* (shown below). Math+Language also builds fluency in all language domains, giving students more confidence in communicating their thinking and understanding of abstract and quantitative concepts. It introduces each math concept abstractly and then provides quantitative examples that are revisited in subsequent activities such as *Word Gallery*.



Matias borrowed \$10,000 at 12% simple interest. How much will he owe the bank in one year? Explain why \$1,299 is the wrong answer. 


$\$10,000 \times .12 + \$99 = \$1,299$ 





Words You Can Use

- + numerator 
- + denominator 
- + principal 
- + interest 
- + simple interest 
- + fee 
- + unit rate 
- + unit price 
- + part-to-part 
- + part-to-whole 


 Listen  Check My Work



Matias borrows \$10,000 to buy the truck. The \$10,000 is .

Matias' bank charges  per month.

After one year, Matias will owe interest of 12% of the \$10,000 principal. The bank also charges a \$99 .

Matias thinks he will owe $\$1,200 + \$99 = \$1,299$. What is he missing?

Matias forgot 

 HINT: What is missing from Matias' equation: the principal, interest, or fee? 

Above: Sentence stems in the "Explain Your Work" math reasoning activity.

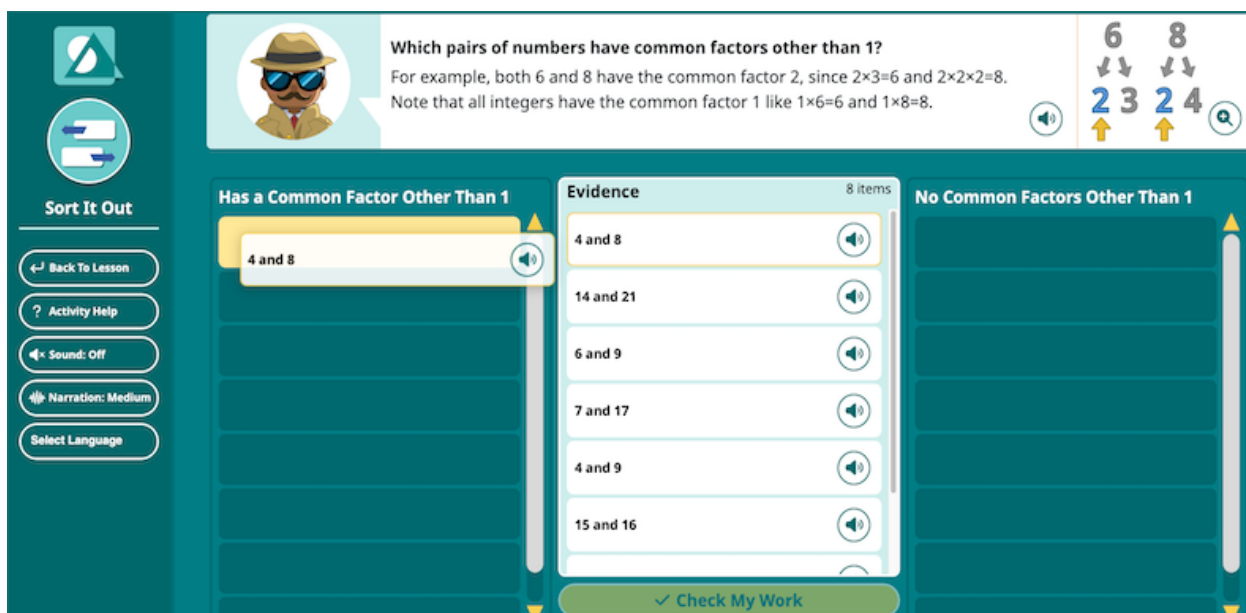


MP3: Construct viable arguments and critique reasoning.

- Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- They can make logical conjectures and explore the truth of their conjectures.
- They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples, justify their conclusions, communicate them to others, and respond to the arguments of others.
- They reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- They are also able to compare two arguments, distinguish correct logic or reasoning from that which is flawed, and can explain any flaws they find.
- Elementary students can construct arguments using concrete referents such as objects, drawings, and diagrams, even if not generalized or made formal.
- Secondary students learn to determine how and where to apply an argument.
- Students at all grades can listen or read the arguments of others, decide if they make sense, and ask questions to clarify or improve the arguments.

MP3: How Speak Agent Helps

Learners use Math+Language to develop knowledge of key math concepts and skills in comprehending and communicating their reasoning. Specifically, learning activities that ask students to weigh and categorize evidence in activities such as *Sort It Out* and in scaffolded writing activities that ask students to explain their thought processes and compare against a model, such as *Explain Your Work*. The program also provides diagrams, illustrations, worked examples, and other referents used to build content knowledge concurrently with language skills.



Above: Evaluating evidence in the "Sort It Out" analysis activity.

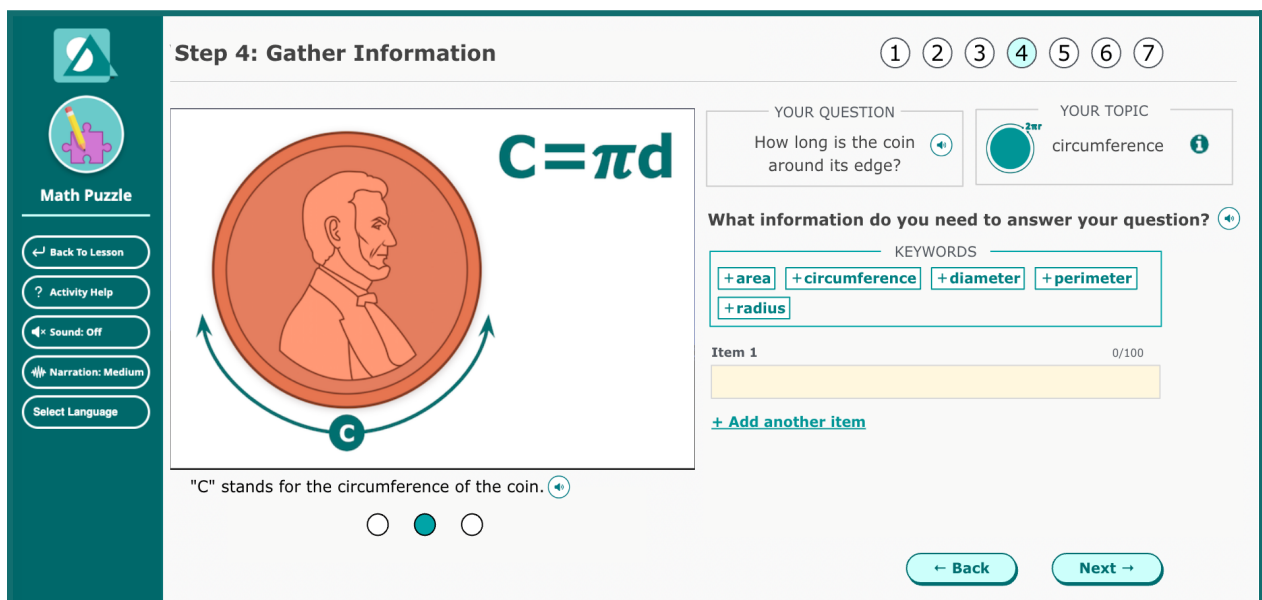


MP4: Model with mathematics.

- Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- In early grades, this may include writing an equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Mathematically proficient students are comfortable making assumptions and approximations to simplify complex situations, returning to revise them later.
- They are able to identify important quantities and map their relationships using tools like diagrams, two-way tables, graphs, flowcharts and formulae.
- They can analyze those relationships mathematically to draw conclusions.
- They can routinely interpret their results in the context of the situation and reflect on whether the results make sense, improving the model if needed.

MP4: How Speak Agent Helps

Math+Language uses authentic stories and problem setups that connect math to everyday problems and situations that are relevant or interesting for the intended student demographic. Our lessons include activities such as *Math Puzzle Maker* wherein students create math problems for classmates to solve. This strategy is vital because it takes students through every step of the math modeling process and provides learning and writing supports at each step.



Step 4: Gather Information

1 2 3 4 5 6 7

YOUR QUESTION
How long is the coin around its edge?

YOUR TOPIC
circumference

What information do you need to answer your question?

KEYWORDS
+ area + circumference + diameter + perimeter
+ radius

Item 1 0/100
[+ Add another item](#)

"C" stands for the circumference of the coin.

← Back Next →

Above: Gathering information in the "Math Puzzle Maker" problem-writing activity.















MP5: Use appropriate tools strategically.

- When solving a problem, mathematically proficient students consider available tools such as concrete models, a ruler, a protractor, a calculator, a spreadsheet, a statistical package, or dynamic geometry software.
- They are familiar with tools appropriate for their grade or course in order to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making math models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- Mathematically proficient students can identify relevant math resources, such as digital content, and use them to pose or solve problems. They can use technological tools to explore and deepen their understanding of concepts.

MP5: How Speak Agent Helps

Math+Language focuses on acquisition of academic language and key math concepts. The stories in our activities model students using the appropriate tools for each particular situation. We also treat the tools themselves as important unit vocabulary that students master during relevant digital lessons.

 point	 line	\overline{AB} line segment	 composite figure
 sketch	 draw 	 straightedge 	 compass 
 geometric construction	 duplicate		



MP6: Attend to precision.

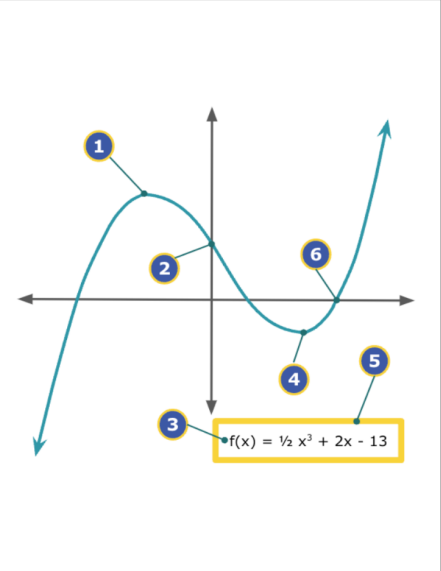
- Mathematically proficient students try to communicate precisely to others.
- They try to use clear definitions in discussion with others and in their own reasoning.
- They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem.
- They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
- In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP6: How Speak Agent Helps

Academic vocabulary comes directly into play with this practice because math terminology tends to be highly precise. Math+Language helps students to develop a precise and nuanced understanding of each math concept and to be able to apply them correctly in varied discourse contexts. Our activities also give students the opportunity to compare related words and to explore their differences. This enhances their understanding of which word to use in a given context.

Label: Characteristics of a Function

Check my Work



function notation	1. maximum
x-intercept	2. y-intercept
	3. _____
function	4. _____
minimum	5. _____
	6. _____

Above: *Connecting related math concepts in the "Diagram It" activity.*

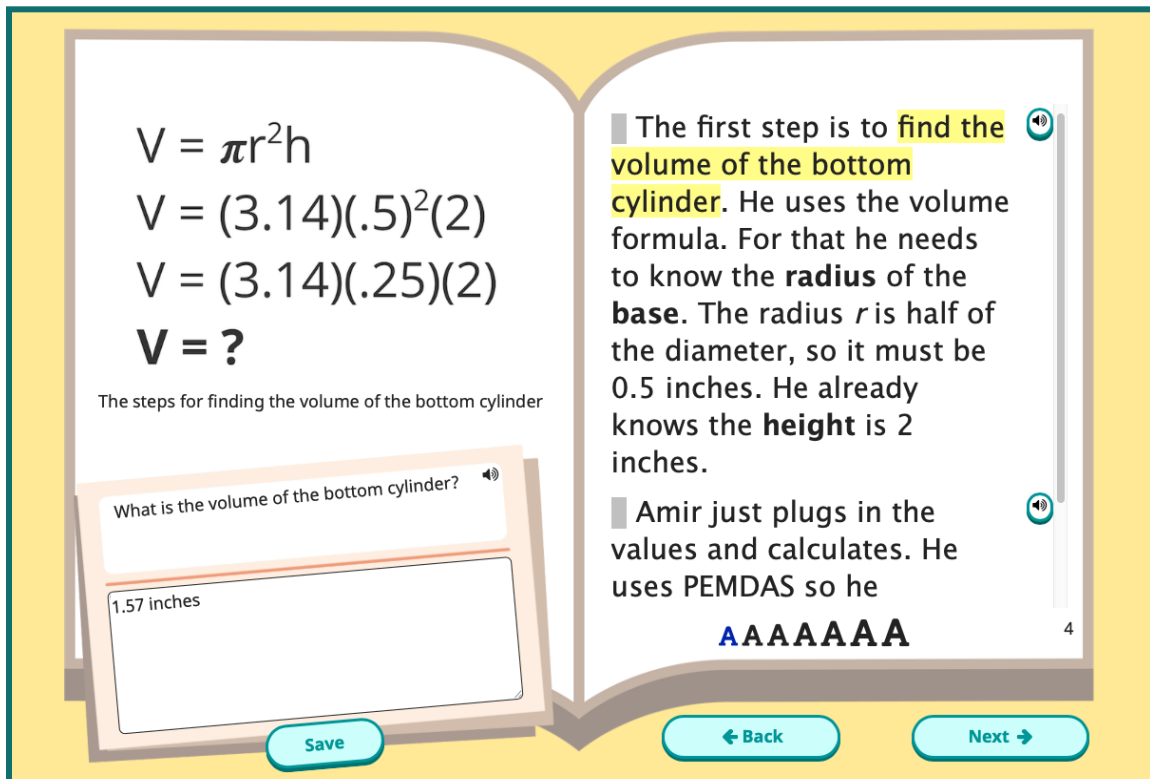


MP7: Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.
- They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- They also can step back for an overview and shift perspective.
- They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

MP7: How Speak Agent Helps

Math+Language introduces new math concepts to students by connecting them to previously learned concepts. When introducing a new mathematical structure such as parts of an equation or the order of operations, our lessons emphasize the relationships among and the use of specific math terminology to help students see the repeated patterns in their math practice in the classroom.



The screenshot shows a math lesson interface. On the left, a list of equations is shown:

$$V = \pi r^2 h$$
$$V = (3.14)(.5)^2(2)$$
$$V = (3.14)(.25)(2)$$
$$V = ?$$

Below the equations, it says "The steps for finding the volume of the bottom cylinder".

On the right, a text box explains the solution:

The first step is to find the volume of the bottom cylinder. He uses the volume formula. For that he needs to know the radius of the base. The radius r is half of the diameter, so it must be 0.5 inches. He already knows the height is 2 inches.

Amir just plugs in the values and calculates. He uses PEMDAS so he

AAAAAAA

At the bottom, there are navigation buttons: "Save", "Back", and "Next".



MP8: Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MP8: How Speak Agent Helps

The Math+Language learning model is built to review the relevant academic language repeatedly throughout each unit. Students are first exposed to the abstract concept and then shown concrete or quantifiable examples numerous times in different authentic contexts. Our digital lessons guide students through the reasoning process, gradually removing learning supports as they progress through the activities.

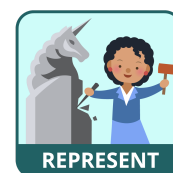
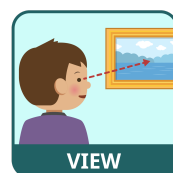
☆ **FOOTNOTE for MP1:**

Specific examples of activities on the Speak Agent platform that model the process of persevering through problem solving:

- The *Tall Tales* activity delivers immediate feedback that allows students to show and practice their reasoning and perseverance.
- *Read Alongs* put math topics into a variety of authentic contexts, requiring students to identify and use evidence from the reading text to respond to the prompts on each page.
- The *Explain Your Work* activity scaffolds the problem solving process with discrete steps that move from gap filling to writing open-ended explanations.



The Six Communication Modes



- **Viewing** is a form of visual learning where students interpret meaning from images, animations, videos, or other graphical elements. This learning mode is used extensively in Speak Agen.
- **Representing** is a form of visual expression where students communicate using graphical elements. Students use this communication mode in Speak Agent to create mathematical problems, models, or to record their observations.

Language in the Mathematical Practices

The table below shows which of the communication modes support each practice:

Math Practices	Listen	Speak	Read	Write	View	Repre-sent
MP1: Make sense of problems and persevere in solving them.	✓		✓		✓	
MP2: Reason abstractly and quantitatively.		✓		✓		✓
MP3: Construct viable arguments and critique reasoning of others.	✓	✓	✓	✓	✓	✓
MP4: Model with mathematics.		✓		✓	✓	✓
MP5: Use appropriate tools strategically.	✓	✓		✓	✓	✓
MP6: Attend to precision.	✓	✓	✓	✓	✓	
MP7: Look for and make use of structure.	✓	✓	✓	✓	✓	✓
MP8: Express regularity in repeated reasoning.			✓	✓	✓	✓

